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Abstract

We resolve redundancies in the characterizations of the Banzhaf value suggested by Haller (1994, Int J Game Theory 23, 261–281) and Malawski (2002, Int J Game Theory 31:47–67). In particular, we show that the collusion properties employed by them are equivalent. Combined with the dummy player axiom, any of the collusion properties has strong symmetry implications whenever the cardinality of the player set exceeds two. Finally, we establish that the Banzhaf value is non-redundantly characterized by the dummy player axiom and any of the collusion properties, provided that the player set is as above.

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Key Words: Banzhaf value, symmetry, collusion, proxy, association, distrust.

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1. Introduction

There are numerous characterizations of the Banzhaf value—first introduced by Banzhaf (1965) for voting games and later extended to general TU games by Owen (1975)—both on fixed and on variable player sets. On the variable ones, the concise characterization by Casajus (2011, Theorem 7) employs just two axioms, the dummy player axiom and some very appealing amalgamation property, 2-efficiency, due to Lehrer (1988).

Let us explain 2-efficiency. When player j is amalgamated to player i in a TU game, j leaves the game as a genuine player, but "sits on the shoulders" of player i, i.e., with respect to the creation of worth, player j is present in a coalition whenever player i is so. 2-efficiency then requires the payoff of player i in the amalgamated game to be the sum of the individual payoffs of players i and j in the original game, i.e., amalgamating players doesn't matter.

Later on, Haller (1994) suggests a collusion property (which we will call proxy neutrality) that breathes the spirit of 2-efficiency, but works on a fixed player set. Instead of leaving the game, player j stays in the game as a null player. Employing proxy neutrality or related collusion properties—association neutrality or distrust neutrality, Haller (1994) and Malawski (2002) suggest characterizations of the Banzhaf value on fixed player sets, which in addition employ the dummy player axiom, symmetry/symmetry invariance, and either linearity or marginality (Young, 1985). Since both authors do not address the redundancy issue and in view of Casajus (2011, Theorem 7), one may be tempted to suspect that one could drop symmetry, linearity, or marginality from these characterizations, at least for the ones that invoke proxy neutrality.

In most cases, this suspicion turns out to be justified. First, we show that the three collusion properties are equivalent (Theorem 1), despite of their differing literal meaning. Further, any of the collusion properties combined with the dummy player axiom already entails symmetry, provided that the player set comprises more than two players (Theorem 2). Building on the former results, we finally establish that the Banzhaf value is non-redundantly characterized by the dummy player axiom and any of the collusion properties, again, on player sets containing at least three players (Theorem 3).

The plan of this paper is as follows: Basic definitions and notation are given in the second section. The third section establishes the relation between the collusion properties. In the fourth section, we explore symmetry implications of the collusion properties. Section five provides our characterizations of the Banzhaf value. Some remarks conclude the paper.

2. Basic definitions and notation

Let $\mathcal{U}$ be a sufficiently large infinite set, the universe of players; $\mathbb{N}(\mathcal{U})$ denotes the set of non-empty and finite set of subsets of $\mathcal{U}$. A **(TU) game** on $\mathcal{U}$ is a pair (N, v) consisting of a set of players $N \in \mathbb{N}(\mathcal{U})$ and a **coalition function** $v \in \mathbb{V}(N) := \{f : 2^N \to \mathbb{R} | f(\emptyset) = 0\}$. Subsets of N are called **coalitions**, and $v(K)$ is called the worth of coalition K. For $v, w \in \mathbb{V}(N)$, $\alpha \in \mathbb{R}$, the coalition functions $v + w \in \mathbb{V}(N)$ and $\alpha \cdot v \in \mathbb{V}(N)$ are given by $(v + w)(K) = v(K) + w(K)$ and $(\alpha \cdot v)(K) = \alpha \cdot v(K)$ for all $K \subseteq N$. For $K \subseteq N$ and $v \in \mathbb{V}(N)$, $v|_K \in \mathbb{V}(K)$ denotes the restriction of v to 2^K. For $T \subseteq N$, $T \neq \emptyset$, the game (N, u_T), $u_T(K) = 1$ if $T \subseteq K$ and $u_T(K) = 0$ otherwise, is called a **unanimity game**. Any $v \in \mathbb{V}(N)$ can be uniquely represented by unanimity games,

$$v = \sum_{T \subseteq N : T \neq \emptyset} \lambda_T(v) \cdot u_T, \qquad \lambda_T(v) := \sum_{S \subseteq T : S \neq \emptyset} (-1)^{|T| - |S|} \cdot v(S). \tag{1}$$

Player $i \in N$ is called a **dummy player** in (N, v) iff $v(K \cup \{i\}) - v(K) = v(\{i\})$ for all $K \subseteq N \backslash \{i\}$; if in addition $v(\{i\}) = 0$, then i is called a **null player**; players $i, j \in N$ are called **symmetric** in (N, v) if $v(K \cup \{i\}) = v(K \cup \{j\})$ for all $K \subseteq N \backslash \{i, j\}$. Let $N_0(v)$ denote the set of null players in (N, v).

A **value** on $N \in \mathbb{N}(\mathcal{U})$ is an operator φ that assigns a payoff vector $\varphi(N, v) \in \mathbb{R}^N$ to any $v \in \mathbb{V}(N)$; a **value** on $\mathbb{N}(\mathcal{U})$ is an operator φ that assigns a payoff vector $\varphi(N, v) \in \mathbb{R}^N$ to any $N \in \mathbb{N}(\mathcal{U})$ and $v \in \mathbb{V}(N)$. For $K \subseteq N$, we set $\varphi_K(N, v) = \sum_{i \in K} \varphi_i(N, v)$. The **Banzhaf value** on N ($\mathbb{N}(\mathcal{U})$) is given by

$$\mathrm{Ba}_i(N, v) = \sum_{K \subseteq N \backslash \{i\}} \frac{v(K \cup \{i\}) - v(K)}{2^{|N|+1}}, \qquad (N \in \mathbb{N}(\mathcal{U})),\ v \in \mathbb{V}(N),\ i \in N. \tag{2}$$

For (N, v) and $i, j \in N$, $i \neq j$, the **amalgamated game** $(N \backslash \{j\}, v_{ij})$ is given by $v_{ij} \in \mathbb{V}(N \backslash \{j\})$,

$$v_{ij}(K) = \begin{cases} v(K \cup \{j\}), & i \in K, \\ v(K), & i \notin K, \end{cases} \qquad K \subseteq N \backslash \{j\}. \tag{3}$$

For (N,v) and $i,j\in N$, $i\neq j$, the **proxy game** $\left(N,v_{ij}^{p}\right)$, the **association game** $\left(N,v_{ij}^{a}\right)$, and the **distrust game** $\left(N,v_{ij}^{d}\right)$ are given by $v_{ij}^{p},v_{ij}^{a},v_{ij}^{d}\in\mathbb{V}(N)$,

$$v_{ij}^{p}(K) = \begin{cases} v(K\cup j), & i\in K,\\ v(K\backslash\{j\}), & i\notin K,\end{cases} \qquad K\subseteq N, \tag{4}$$

$$v_{ij}^{a}(K) = \begin{cases} v(K\cup\{i,j\}), & K\cap\{i,j\}\neq\emptyset,\\ v(K), & K\cap\{i,j\}=\emptyset,\end{cases} \qquad K\subseteq N, \tag{5}$$

$$v_{ij}^{d}(K) = \begin{cases} v(K), & \{i,j\}\subseteq K,\\ v(K\backslash\{i,j\}), & \{i,j\}\nsubseteq K,\end{cases} \qquad K\subseteq N, \tag{6}$$

respectively.

Below, we list the axioms that are used later on. Unless made explicit, these axioms are supposed to hold for fixed $N,N'\in\mathbb{N}(\mathcal{U})$.

Linearity, L. For all $v,w\in\mathbb{V}(N)$ and $\alpha\in\mathbb{R}$, $\varphi(N,v+w)=\varphi(N,v)+\varphi(N,w)$ and $\varphi(N,\alpha\cdot v)=\alpha\cdot\varphi(N,v)$.

Null player, N. For all $v\in\mathbb{V}(N)$ and all $i\in N$, who are null players in (N,v), $\varphi_i(N,v)=0$.

Null player out, NPO. For all $N\in\mathbb{N}(\mathcal{U})$, $v\in\mathbb{V}(N)$, all $i\in N$, who are null players in (N,v), and all $j\in N\backslash\{j\}$, we have $\varphi_j\left(N\backslash\{i\},v|_{N\backslash\{i\}}\right)=\varphi_j(N,v)$.

Dummy player, D. For all $v\in\mathbb{V}(N)$ and all $i\in N$, who are dummy players in (N,v), $\varphi_i(N,v)=v(\{i\})$.

Symmetry, S. For all $v\in\mathbb{V}(N)$ and all $i,j\in N$, who are symmetric in (N,v), $\varphi_i(N,v)=\varphi_j(N,v)$.

Isomorphism invariance, II. For all $N,N'\in\mathbb{N}(\mathcal{U})$, any bijection $\pi:N\rightarrow N'$, and $v\in\mathbb{V}(N)$, we have $\varphi_{\pi(i)}(N',v\circ\pi^{-1})=\varphi_i(N,v)$ for all $i\in N$, where $v\circ\pi^{-1}\in\mathbb{V}(N')$ is given by $(v\circ\pi^{-1})(K')=v(\pi^{-1}(K'))$, $K'\subseteq N'$.

Symmetry invariance, SI. For all $v\in\mathbb{V}(N)$, $i\in N$, and all bijections $\pi:N\rightarrow N$, $\varphi_{\pi(i)}(N,v\circ\pi^{-1})=\varphi_i(N,v)$.

Marginality, M. For all $v,w\in\mathbb{V}(N)$ and all $i\in N$ such that $v(K\cup\{i\})-v(K)$ $=w(K\cup\{i\})-w(K)$ for all $K\subseteq N\backslash\{i\}$, $\varphi_i(N,v)=\varphi_i(N,w)$.

2-Efficiency, 2E. For all $N\in\mathbb{N}(\mathcal{U})$, $v\in\mathbb{V}(N)$, and $i,j\in N$, $i\neq j$, $\varphi_i(N\backslash\{j\},v_{ij})$ $=\varphi_i(N,v)+\varphi_j(N,v)$.

Proxy neutrality, PN. For all $v \in \mathbb{V}(N)$ and $i, j \in N$, $i \neq j$, $\varphi_i\left(N, v_{ij}^p\right) + \varphi_j\left(N, v_{ij}^p\right) = \varphi_i(N, v) + \varphi_j(N, v)$.

Association neutrality, AN. For all $v \in \mathbb{V}(N)$ and $i, j \in N$, $i \neq j$, $\varphi_i\left(N, v_{ij}^a\right) + \varphi_j\left(N, v_{ij}^a\right) = \varphi_i(N, v) + \varphi_j(N, v)$.

Distrust neutrality, DN. For all $v \in \mathbb{V}(N)$ and $i, j \in N$, $i \neq j$, $\varphi_i\left(N, v_{ij}^d\right) + \varphi_j\left(N, v_{ij}^d\right) = \varphi_i(N, v) + \varphi_j(N, v)$.

3. Relation between the collusion properties

Despite their structural similarity, the immediate economic content of the three collusion properties is quite different. **PN** requires two players' joint payoff not to be affected when their economic power is shifted to one of them—their proxy, while the other becomes completely powerless. In contrast, **AN** and **DN** treat the colluding players symmetrically. Under the association agreement of **AN** embodied in (5), any of them alone is as productive as they jointly are, whereas under the distrust agreement of **DN** due to (6), both players alone are completely unproductive, while their joint economic force remains unaffected.

The following theorem reveals that these collusion properties ultimately entail the same economic implications. The reason for this equivalence seems to be the that they all impose similar and interrelated consistency requirements on values. Later on, this fact turns out to be useful in extending claims involving **D** and **PN** to related claims that invoke **AN** or **DN** instead of **PN**. Note that **PN** invites the application of **D** (or just **N**), while the other collusion properties do not so. Generically, **PN** turns non-null players into null-players, which can be handled by **D.**

Theorem 1. ***PN**, **AN**, and **DN** are equivalent.*

Proof. (i) **AN** implies **PN** and **DN**. Let φ on $N \in \mathbb{N}(\mathcal{U})$ obey **AN**. By (4), (5) and (6), we have $\left(v_{ij}^p\right)_{ij}^a = v_{ij}^a$ and $\left(v_{ij}^d\right)_{ij}^a = v_{ij}^a$ for all $i, j \in N$ and $v \in \mathbb{V}(N)$. The former entails

$$\begin{aligned} \varphi_i\left(N, v_{ij}^p\right) + \varphi_j\left(N, v_{ij}^p\right) &\overset{\mathbf{AN}}{=} \varphi_i\left(N, \left(v_{ij}^p\right)_{ij}^a\right) + \varphi_j\left(N, \left(v_{ij}^p\right)_{ij}^a\right) \\ &= \varphi_i\left(N, v_{ij}^a\right) + \varphi_j\left(N, v_{ij}^a\right) \overset{\mathbf{AN}}{=} \varphi_i(N, v) + \varphi_j(N, v). \end{aligned}$$

Hence, φ obeys **PN**. Analogously for **DN**.

(ii) **PN** as well as **DN** imply **AN**. By (4), (5), and (6), we have $\left(v_{ij}^{a}\right)_{ij}^{p} = v_{ij}^{p}$ and $\left(v_{ij}^{a}\right)_{ij}^{d} = v_{ij}^{d}$ for all $i, j \in N$ and $v \in \mathbb{V}(N)$. The proof now continues as in (i), respectively. $\square$

4. Collusion properties and symmetry

Casajus (2011, Theorem 1) establishes that **2E** has strong symmetry implications. In particular, **2E** implies **II**, hence, **SI** and **S** (all on $\mathbb{N}(\mathcal{U})$). Since **2E** and **PN** breathe the same spirit, **PN** may entail similar symmetry properties.

Of course, **II** is out of reach because **PN** applies to a fixed player set. Yet, **PN** alone is not powerful enough to trigger **SI** or just **S**, except, of course, when they have no implications at all, i.e., for $|N| = 1$. For $|N| > 1$, fix some non-constant mapping $\xi : N \to \mathbb{R}$ and consider the value $\varphi^{(1)}$ on N given by

$$\varphi_i^{(1)}(N, v) = \mathrm{Ba}_i(N, v) + \xi(i), \qquad i \in N,\ v \in \mathbb{V}(N). \tag{7}$$

While $\varphi^{(1)}$ inherits **PN** from Ba, this does not hold true for **SI** or **S**.

Note that $\varphi^{(1)}$ violates **D**. But even **PN** and **D** together do not enforce **SI** or **S** for $|N| = 2$, but otherwise they do so. To see the former, consider the value $\varphi^{(2)}$ on $N = \{1, 2\}$ given by

$$\varphi_1^{(2)}(N, v) = v(\{1\}) \quad \text{and} \quad \varphi_2^{(2)}(N, v) = v(N) - v(\{1\}), \qquad v \in \mathbb{V}(N), \tag{8}$$

which satisfies both **PN** and **D**, but obviously fails **SI** and **S.**

Theorem 2. *If $|N| \neq 2$, then **(PN** or **AN** or **DN)** and **D** imply **S.***

Proof. By Theorem 1, it suffices to show this for **PN**. For $|N| = 1$, nothing is to show. Let now $|N| > 2$ and let φ on N obey **PN** and **D**. Further, let i and j be symmetric in (N, v). Since $|N| > 2$, there is some $k \in N \backslash \{i, j\}$. By (4), **PN**, and **D**, we have

$$\varphi_k\left(N, v_{ki}^{p}\right) - \varphi_i(N, v) = \varphi_k(N, v) = \varphi_k\left(N, v_{kj}^{p}\right) - \varphi_j(N, v). \tag{9}$$

Moreover, (4), **PN**, and **D** imply

$$\begin{aligned} \varphi_k\left(N, v_{ki}^{p}\right) + \varphi_j\left(N, v_{ki}^{p}\right) &= \varphi_k\left(N, \left(v_{ki}^{p}\right)_{kj}^{p}\right) \\ &= \varphi_k\left(N, \left(v_{kj}^{p}\right)_{ki}^{p}\right) = \varphi_k\left(N, v_{kj}^{p}\right) + \varphi_i\left(N, v_{kj}^{p}\right), \end{aligned} \tag{10}$$

where the second equation drops from (4) entailing $(v^p_{ki})^p_{kj} = \left(v^p_{kj}\right)^p_{ki}$. Again by (4), **PN**, and **D**, we have

$$\varphi_j\left(N, v^p_{ki}\right) = \varphi_i\left(N, (v^p_{ki})^p_{ij}\right). \tag{11}$$

Further, (4) and the assumption that i and j are symmetric in (N, v) show

$$v^p_{kj} = (v^p_{ki})^p_{ij}. \tag{12}$$

Finally, (9)–(12) together yield $\varphi_i(N, v) = \varphi_j(N, v)$. □

Remark 1. Malawski (2002, Section 6) did a small step towards Theorem 2 by considering relaxations of **S**.

5. Collusion properties and the Banzhaf value

Only recently, Casajus (2011, Theorem 7) shows that the Banzhaf value (on $\mathbb{N}(\mathcal{U})$) is characterized by **D** and **2E**, entailing that the characterization by Nowak (1997, Theorem) via **D**, **2E**, **S**, and **M** is redundant. Since **PN** is in the spirit of **2E** and by Theorem 1, one might curious whether the same holds true for the characterizations by Haller (1994, Propositions 7 and 8) and Malawski (2002, Theorem 3 and Corollary 2), which besides one of the collusion properties and **D** employ **S**/**SI** and either **L** or **M**. By Theorems 1 and 2, and Malawski (2002, Corollary 2), we already know that **S**/**SI** can be dropped whenever $|N| \neq 2$. Yet, we have even more.

Theorem 3. *If $|N| \neq 2$, then the Banzhaf value on N is the unique value that satisfies (**PN** or **AN** or **DN**) and **D**.*

Proof. In view of Theorem 1, it suffices to show this for **PN**. By Haller (1994, Proposition 7) and (2), Ba on N meets **PN** and **D**, respectively. Remains to deal with uniqueness. Let φ obey **PN** and **D**. For $|N| = 1$, the claim is immediate from **D**. Let now $|N| > 2$. We proceed by induction on $|N \backslash N_0(v)|$.

Induction basis: For $v \in \mathbb{V}(N)$, $|N \backslash N_0(v)| < 2$, $\varphi(N, v) = \mathrm{Ba}(N, v)$ holds by **D**. Consider now $v \in \mathbb{V}(N)$, $|N \backslash N_0(v)| = 2$. Again by **D**, $\varphi_i(N, v) = \mathrm{Ba}_i(N, v)$ for $i \in N_0(v)$. W.l.o.g., let $N \backslash N_0(v) = \{1, 2\}$ and $3 \in N_0(v)$. By (1), we have

$$v = \lambda_1 \cdot u_{\{1\}} + \lambda_2 \cdot u_{\{2\}} + \lambda_{12} \cdot u_{\{1,2\}}, \qquad \lambda_1, \lambda_2, \lambda_{12} \in \mathbb{R}.$$

Let $w \in \mathbb{V}(N)$,

$$w = (\lambda_1 - \lambda_2) \cdot u_{\{3\}} + \lambda_2 \cdot \left(u_{\{1\}} + u_{\{2\}}\right) + \lambda_{12} \cdot u_{\{1,2\}}.$$

By (4), $v = w_{13}^p$ and $w_{12}^p = (\lambda_1 - \lambda_2) \cdot u_{\{3\}} + (2 \cdot \lambda_2 + \lambda_{12}) \cdot u_{\{1\}}$. Further, 3 and 1 are dummy players in (N, w) and (N, w_{12}^p), respectively. By (4), **PN**, and **D**, we have

$$\varphi_1(N, w) + \lambda_1 - \lambda_2 = \varphi_1(N, w) + \varphi_3(N, w) = \varphi_1(N, w_{13}^p) = \varphi_1(N, v) \qquad (13)$$

and

$$\varphi_1(N, w) + \varphi_2(N, w) = \varphi_1(N, w_{12}^p) = 2 \cdot \lambda_2 + \lambda_{12}. \qquad (14)$$

By Theorem 2, φ meets **S**. Since i and j are symmetric in (N, w), (13) and (14) already imply

$$\varphi_1(N, v) = \lambda_1 + \frac{\lambda_{12}}{2} = \mathrm{Ba}_1(N, v). \qquad (15)$$

By (4), $v_{12}^p = (\lambda_1 + \lambda_2 + \lambda_{12}) \cdot u_{\{1\}}$, and 1 is a dummy player in (N, v_{12}). Thus, (4), **PN**, and **D** entail

$$\varphi_1(N, v) + \varphi_2(N, v) = \varphi_1(N, v) = \lambda_1 + \lambda_2 + \lambda_{12}. \qquad (16)$$

Finally, (15) and (16) yield

$$\varphi_2(N, v) = \lambda_2 + \frac{\lambda_{12}}{2} = \mathrm{Ba}_2(N, v).$$

Induction hypothesis: Suppose $\varphi(N, v) = \mathrm{Ba}(N, v)$ for $v \in \mathbb{V}(N)$, $|N \backslash N_0(v)| \leq k$, $k \in \mathbb{N}$, $k \geq 2$.

Induction step: Let $v \in \mathbb{V}(N)$, $|N \backslash N_0(v)| = k + 1$. By **D,** $\varphi_i(N, v) = \mathrm{Ba}_i(N, v)$ for $i \in N_0(v)$. By **PN** and **D**, $(\varphi_i(N, v))_{i \in N \backslash N_0(v)}$ is a solution to the following system of linear equations

$$\varphi_i(N, v) + \varphi_j(N, v) = \varphi_i\left(N, v_{ij}^p\right), \qquad (i, j) \in N \backslash N_0(v) \times N \backslash N_0(v), \ i \neq j. \quad (17)$$

By (4), $\left|N \backslash N_0\left(v_{ij}^d\right)\right| < |N \backslash N_0(v)|$. Hence, by the induction hypothesis, the right-hand side of (17) is determined by Ba. Since Ba meets **PN** and **D,** $(\mathrm{Ba}_i(N, v))_{i \in N \backslash N_0(v)}$ is a solution to (17), which is unique because of $|N \backslash N_0(v)| \geq 3$. $\square$

Remark 2. Note that the proof of Theorem 3 rests on ideas of the proofs of Lehrer (1988, Remark 3) and Casajus (2011, Theorem 7), the former in the induction step and the latter in the crucial part of the induction basis. Embarrassingly, the use of Theorem 2 in this proof indicates that the proofs of Casajus (2011, Theorem 7 and Corollary 8) can be considerably simplified by using Casajus (2011, Theorem 1) :(

Remark 3. For $|N| > 2$, our characterizations are non-redundant. The Shapley (1953) value meets **D**, but fails **PN**, **AN**, and **DN**. The value $\varphi^{(1)}$ from (7) obeys **PN**, **AN**, and **DN**, but fails **D**.

Remark 4. Theorem 3 fails for $|N| = 2$. Consider the values $\varphi^{(2)} \neq \mathrm{Ba}$ from (8) and $\varphi^{(3)} \neq \mathrm{Ba}$, w.l.o.g., on $N = \{1, 2\}$, the latter given by

$$\varphi_i^{(3)}(N, v) = \begin{cases} v(\{i\}) + \dfrac{v(\{i\})}{v(\{1\}) + v(\{2\})} \cdot \lambda_N(v), & v(\{1\}) + v(\{2\}) \neq 0, \\ v(\{i\}) + \frac{1}{2} \cdot \lambda_N(v), & v(\{1\}) + v(\{2\}) = 0, \end{cases}$$

$i \in N$, $v \in \mathbb{V}(N)$, which both satisfy **D**, **PN**, **AN**, and **DN**. Moreover, it doesn't help to add only one of the following axioms: **L, M** or **S/SI**. Just observe that $\varphi^{(2)}$ meets **L** and **M**, but not **S/SI**, whereas $\varphi^{(3)}$ obeys **S/SI** but neither **L** nor **M**. Hence, the Haller (1994, Propositions 7 and 8) and Malawski (2002, Theorem 3 and Corollary 2) characterizations are non-redundant for $|N| = 2$.

6. Conclusion

The main point of this paper is that the collusion properties suggested by Haller (1994) and Malawski (2002) are equivalent and have stronger implications then observed so far. These implications resemble those of **2E**, recently discovered by Casajus (2011). Quite naturally, this triggers the question on the relation between **2E** and the collusion properties.

Theorem 4. ***2E*** *implies* ***PN, AN,*** *and* ***DN*** *on* $\mathbb{N}(\mathcal{U})$.

Proof. Let φ obey **2E**. For $N \in \mathbb{N}(\mathcal{U})$, $|N| = 1$, nothing is to show. By (3) and (4), we have $\left(v_{ij}^p\right)_{ij} = v_{ij}$ for all $N \subseteq \mathbb{N}(\mathcal{U})$, $|N| > 1$, $v \in \mathbb{V}(N)$, and $i, j \in N$. This already entails

$$\begin{aligned} \varphi_i\left(N, v_{ij}^p\right) + \varphi_j\left(N, v_{ij}^p\right) &\overset{\mathbf{2E}}{=} \varphi_i\left(N \backslash \{j\}, \left(v_{ij}^p\right)_{ij}\right) \\ &= \varphi_i\left(N \backslash \{j\}, v_{ij}\right) \overset{\mathbf{2E}}{=} \varphi_i(N, v) + \varphi_j(N, v), \end{aligned}$$

i.e., φ meets **PN** on N. In view of Theorem 1, this already completes the proof. $\square$

Of course, there is no hope to have a strong counterpart to Theorem 4 in the opposite direction. First, the collusion properties apply to a fixed player set. And second, other than **2E**, **PN** keeps the null player generated by the proxy agreement. While invoking **NPO** may remedy to the former obstacle, the second one can be dealt with by **N**. The obvious proof is left to the reader. Further, it is not too difficult to check that one cannot do without **NPO** or **N**.

References

Banzhaf, J. F. (1965). Weighted voting does not work: A mathematical analysis, *Rutgers Law Review* **19**: 317–343.

Casajus, A. (2011). Amalgamating players, symmetry, and the Banzhaf value, *International Journal of Game Theory* (forthcoming). doi: 10.1007/s00182-011-0300-z.

Haller, H. (1994). Collusion properties of values, *International Journal of Game Theory* **23**: 261–281.

Lehrer, E. (1988). An axiomatization of the Banzhaf value, *International Journal of Game Theory* **17**(2): 89–99.

Malawski, M. (2002). Equal treatment, symmetry and Banzhaf value axiomatizations, *International Journal of Game Theory* **31**: 47–67.

Nowak, A. S. (1997). On an axiomatization of the Banzhaf value without the additivity axiom, *International Journal of Game Theory* **26**: 137–141.

Owen, G. (1975). Multilinear extensions and the Banzhaf value, *Naval Research Logistic Quarterly* **22**: 741–750.

Shapley, L. S. (1953). A value for n-person games, *in* H. Kuhn and A. Tucker (eds), *Contributions to the Theory of Games*, Vol. II, Princeton University Press, Princeton, pp. 307–317.

Young, H. P. (1985). Monotonic solutions of cooperative games, *International Journal of Game Theory* **14**: 65–72.

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